Solid substitution: theory versus experiment

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SUMMARY

Gassmann fluid substitution is widely used in geophysical practice. In the last few years, the topic of fluid/solid substitution has emerged, where the substances filling the pore space can be solids, fluids, or visco-elastic materials, such as heavy oils. Solid substitution cannot be accomplished with the Gassmann theory because the finite rigidity of the pore fill (either solid or viscoelastic) prevents pressure communication throughout the pore space, which is a key assumption of the Gassmann theory. In this paper we explore applicability of solid substitution techniques by using a sandstone saturated with a solid substance, octodecane. This substance is a hydrocarbon with a melting point of 28°C, making it convenient to use in the lab in both solid and fluid form. Our approach is to measure a dry sandstone sample under different confining pressure, then saturate it with liquid Octodecane at 35°C and measure again. After that, we cool it to 20-25°C and carry out the measurement once more. The dry properties can be used to obtain parameters necessary for fluid and solid substitution. The results show that moduli of the dry sandstone exhibit significant pressure dependency, which is reduced for the solid filled rock. Also the prediction of the Gassmann theory and Ciz and Shapiro theory underestimate the velocities. This suggests that stiffening occurs due to substantial reduction of compliance of grain contacts by the solid infill. This effect is accounted for by the solid squirt theory. The results give direct evidence of the solid squirt effect and can be used to verify and calibrate theoretical solutions for rocks saturated with solid or viscoelastic substances.

Key words: Rock physics, Fluid/Solid substitution, Gassmann's theory, Squirt theory.

INTRODUCTION

Fluid substitution using Gassmann equation (1951) is widely employed in geophysical practice. It is proved to be an effective way to predict the bulk modulus of an isotropic homogeneous rock saturated with ideal fluid under quasi-static condition. For the last decade, the topic of solid substitution has emerged, where the substances saturated in the pore space are replaced with real solids or viscoelastic materials, such as heavy oil. It's believed that solid substitution plays a significant role in the characterization of salt precipitation, rock matrix dissolution, and other related problems. However, solid substitution cannot be accomplished with the Gassmann equation because the presence of the finite rigidity of the pore fill (either solid or viscoelastic) prevents the pressure equilibration throughout the pore space, which is a key assumption of the Gassmann theory.

Ciz and Shapiro (2007) derived the exact solid substitution equation. However, it contains a new defined heuristic parameter, which is difficult to estimate. Hence, they proposed a simplified equation, which reduces to Gassmann equation for the bulk modulus, and a similar equation for the shear modulus. Saxena and Mavko (2014) derived exact solid-to-solid substitution equations using an alternative approach: reciprocity based on the dual porosity structure. However, the scheme is not applicable when the pore filling material is ideal fluid. Thus, Gurevich and Saxena (2015) proposed a modified model by replacing the P-wave modulus of the inclusion L_f with Young's modulus E_f. Nevertheless, Sayers (2015) pointed out that this simple model is expected to underestimate the unrelaxed frame moduli for small shear modulus of the pore filling material μ_f due to the fact that it works only if the thickness of the interlayer is large compared with its diameter according to the definition of E_f. However, it is always the opposite case for typical compliant pores. Hence, Glubokovskikh and Gurevich (2016) employed the solution of Tsai and Lee (1998) to modify the Gurevich and Saxena (2015) model, which established a more accurate model considering the dependence of the effective compressional stiffness of the inclusion M_f on its shear modulus μ_f.

In this paper, we explore the applicability and validity of these developed solid substitution schemes through a sample of sandstone saturated with a solid substance called Octodecane. This substance is a hydrocarbon with a melting point of 28°C, which makes it convenient to use in the lab in both solid and fluid form. The elastic properties of this sample are then both measured in the lab and predicted using these theoretical schemes. Hence, the applicability and validity of these schemes can be analysed by comparing the theoretical predictions to the experimental results.

METHOD

Ciz and Shapiro (2007) derived the simplified expressions for the bulk and shear moduli of the solid saturated isotropic rocks as follows:

$$\frac{K_{sat}}{K_g-K_{sat}} = \frac{K_{dry}}{K_g-K_{dry}} + \frac{K_f}{\phi(K_g-K_f)}, \quad (1)$$

$$\frac{\mu_{sat}}{\mu_g-\mu_{sat}} = \frac{\mu_{dry}}{\mu_g-\mu_{dry}} + \frac{\mu_f}{\phi(\mu_g-\mu_f)}, \quad (2)$$

where $\boldsymbol{K_{sat}}$ and $\boldsymbol{\mu_{sat}}$ are the effective bulk and shear moduli of the solid filled rock, respectively. $\boldsymbol{K_f}$ and $\boldsymbol{\mu_f}$ are the bulk and shear moduli of the pore-filling material (solid or fluid), respectively. $\boldsymbol{K_{dry}}$ and $\boldsymbol{\mu_{dry}}$ are the bulk and shear moduli of the dry rock, respectively. $\boldsymbol{\phi}$ is the porosity.

Following Gurevich et al. (2009) and using the formalism proposed by Sayers and Kachanov (1991, 1995), Saxena and Mavko (2014) proposed the expression of the unrelaxed frame stiffness in place of the dry rock moduli. Hence, the rock stiffening due to the squirt flow and shear dispersion (solid squirt) effects is considered as follows:

$$\frac{1}{K_{uf}} = \frac{1}{K_h} + \frac{1}{\frac{1}{\frac{1}{K_{dry}}-\frac{1}{K_h}}+\frac{L_f}{\phi_c}}, \quad (3)$$

$$\frac{1}{\mu_{uf}} = \frac{1}{\mu_h} + \frac{4}{15}\frac{1}{\frac{1}{\frac{1}{K_{dry}}-\frac{1}{K_h}}+\frac{L_f}{\phi_c}} + \frac{1}{\frac{1}{\frac{1}{\mu_{dry}}-\frac{1}{\mu_h}-\frac{4}{15}\left(\frac{1}{K_{dry}}-\frac{1}{K_h}\right)}+\frac{5\mu_f}{2\phi_c}}, \quad (4)$$

where $\boldsymbol{K_{uf}}$ and $\boldsymbol{\mu_{uf}}$ are the unrelaxed frame stiffness, respectively. $\boldsymbol{K_h}$ and $\boldsymbol{\mu_h}$ are the high pressure limit moduli with all compliant pores closed. $\boldsymbol{K_{dry}}$ and $\boldsymbol{\mu_{dry}}$ are the pressure dependent bulk and shear moduli of the dry rock, respectively. $\boldsymbol{L_f}$ is the P-wave modulus of the pore filling material. $\boldsymbol{\phi_c}$ is the compliant porosity.
The remaining stiff pores are then saturated with the solid by using the lower embedded bound method by Mavko and Saxena (2013). Hence, the undrained rock bulk and shear moduli can be predicted as follows:

$$K_{sat} = K_{bc} + \frac{\left(1-\frac{K_{bc}}{K_g}\right)^2}{\frac{\phi_s}{K_f}+\frac{1-\phi_s}{K_g}-\frac{K_{bc}}{(K_g)^2}}, \quad (5)$$

$$\mu_{sat} = \mu_{bc} + \frac{\left(1-\frac{\mu_{bc}}{\mu_g}\right)^2}{\frac{\phi_s}{\mu_f}+\frac{1-\phi_s}{\mu_g}-\frac{\mu_{bc}}{(\mu_g)^2}}, \quad (6)$$

where

$$K_{bc} = \frac{(1-\phi_s)\left(\frac{1}{K_g}-\frac{1}{K_{uf}}\right)+\frac{3\phi_s}{4}\left(\frac{1}{\mu_g}-\frac{1}{\mu_{uf}}\right)}{\frac{1}{K_g}\left(\frac{1}{K_g}-\frac{1}{K_{uf}}\right)+\frac{3\phi_s}{4}\left(\frac{1}{K_g\mu_g}-\frac{1}{K_{uf}\mu_{uf}}\right)}, \quad (7)$$

$$\mu_{bc} = \frac{(1-\phi_s)\left(\frac{1}{\mu_g}-\frac{1}{\mu_{uf}}\right)+\frac{3\phi_s}{4}\left(\frac{1}{\chi_g}-\frac{1}{\chi_f}\right)}{\frac{1}{\mu_g}\left(\frac{1}{\mu_g}-\frac{1}{\mu_{uf}}\right)+\frac{3\phi_s}{4}\left(\frac{1}{\chi_g\mu_g}-\frac{1}{\chi_{uf}\mu_{uf}}\right)}, \quad (8)$$

$$\chi = \frac{\mu(9K+8\mu)}{8(K+2\mu)}, \quad (9)$$

$\boldsymbol{\phi_s}$ is the remaining stiff porosity.
As aforementioned, Gurevich and Saxena (2015) used $\boldsymbol{E_f}$ to replace the term $\boldsymbol{L_f}$. In that case, equations 5 and 6 can reduce to Gassmann equation when the filling material is ideal fluid. Furthermore, Glubokovskikh and Gurevich (2016) established a more accurate model recently, which expressed the effective compressional stiffness of the inclusion $\boldsymbol{M_f}$ as a function of its shear modulus $\boldsymbol{\mu_f}$ as follows:

$$\boldsymbol{M_f = K_f + \mu_f - \frac{\left(K_f-\frac{2}{3}\mu_f\right)^2}{\left(K_f+\frac{4}{3}\mu_f\right)\frac{\alpha\gamma\cdot I_0(\alpha\gamma_f)}{2I_1(\alpha\gamma_f)}-\mu_f}}, \quad (10)$$

where $\gamma_f = \sqrt{\frac{36\mu_f}{3K_f+4\mu_f}}$, $\alpha = r/h$ is an inverse aspect ratio, and I_k denotes a modified Bessel function of the first kind and of the order k.

Octodecane experiment

As an example, we examine the validity and applicability of the aforementioned solid substitution schemes with a sample of sandstone saturated with a solid substance. This substance is a type of hydrocarbon called Octodecane. And its melting point is 28°C, which makes it convenient to use in the lab in both solid and fluid form. Our approach is to measure the dry sandstone sample at different confining pressure first. Then, we saturate it with liquid Octodecane at 35°C and measure again. After that, we cool it to 20-25°C and carry out the measurement once more. The dry rock moduli as a function of confining pressure can be used to obtain the required parameters for fluid and solid substitution (Shapiro, 2003). For the Octodecane in fluid form, we use 1.42GPa for its bulk modulus at temperature 38°C, while a bulk modulus of 3.79 GPa and a shear modulus of 1.13 GPa are used for the solid form, along with a density of 0.78 g/cm^3. The other properties of the sample are as follows: porosity: 23.56%, grain density: 2.58 g/cm^3, mineral bulk modulus 37 GPa, shear modulus: 45 GPa.

In Figure 1a and 1b, we compare the bulk and shear moduli predictions under different confining pressures given by several solid substitution schemes. Figure 2a and 2b show the similar plots for P-wave and S-wave velocities. The elastic moduli of the dry and Octodecane (both in fluid and solid form) saturated sandstone sample measured at the ultrasonic frequency are shown as blue open circles, blue filled squares and black filled circles, respectively. For the fluid substitution, we note that the measurement results have a good agreement with those predicted by the Gassmann theory for the bulk modulus. However, a small discrepancy between them is found for the shear modulus (red x-marks). For the solid substitution, the predictions by Ciz and Shapiro (2007) model (black stars) considerably underestimate the elastic moduli compared to the experimental results. This is due to the stiffening effect by the solid squirt, which is not considered in the model of Ciz and Shapiro. Similar to the fluid squirt flow, the solid squirt is induced by the heterogeneous stress distributions in the pore space. To take into account this effect, the binary pore structure is usually employed, which divides the pore space into compliant and stiff pores. This is done in the models of Saxena and Gurevich (2015), Gurevich and Saxena (2015), and Glubokovskikh and Gurevich (2016), as shown in the last section. However, we can observe in Figure 1a and 1b that these three models give almost the same estimates close to those given by the lower embedded bound theory (Saxena and Mavko, 2013), which defines the smallest change upon solid substitution. Hence, they also underestimate the solid squirt effect. Nevertheless, their results are much better than Ciz and Shapiro (2007) model. This indicates that only considering the compliant pores might not fully describe the solid squirt effects.

A similar problem was studied by Paula et al. (2012), who found that the characteristic frequency of typical compliant pores lies close to the seismic frequency domain, but is well below the ultrasonic frequencies. Hence, they argued that the solid squirt occurs from the pores, which has an aspect ratio larger than that of the compliant pores, but much smaller than that of stiff pores. This provides a good prospective to better explain the solid squirt effect. We will study this in detail in the future.

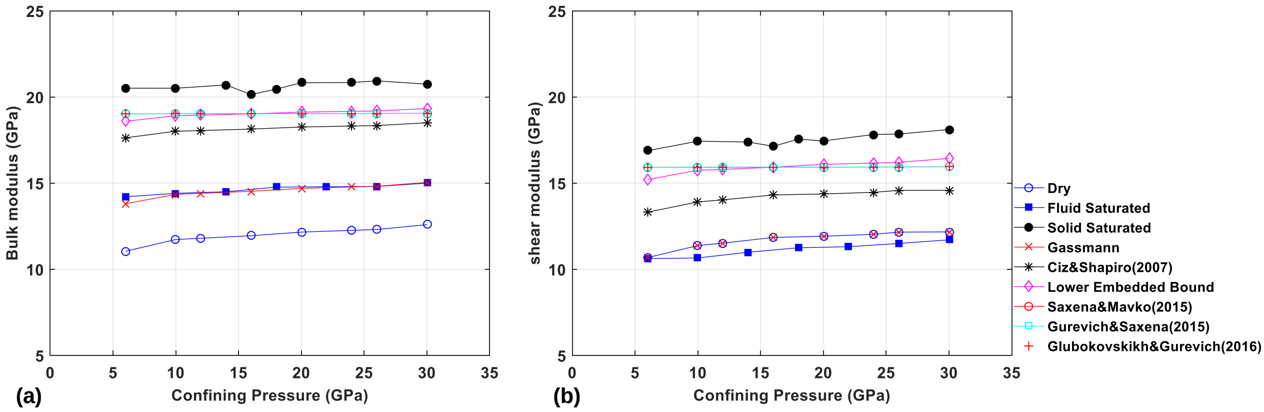

Figure 1. Comparison of the theoretical estimates of bulk (a) and shear (b) moduli of octodecane saturated sandstone using different solid substitution schemes with the experiment as a function of confining pressure.

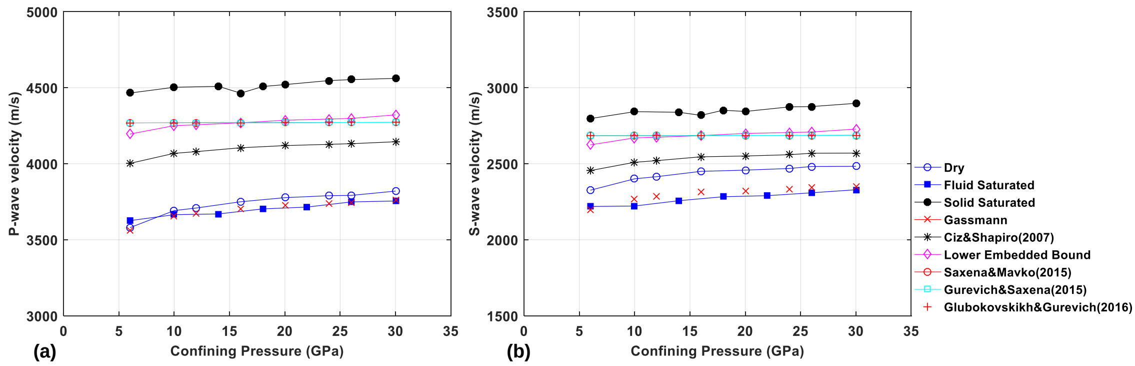

Figure 2. Comparison of the theoretical estimates of P-wave (a) and S-wave (b) velocity of octodecane saturated sandstone using different solid substitution schemes with the experiment as a function of confining pressure.

CONCLUSIONS

The experiment reveals that moduli of dry rock exhibits significant pressure dependency, which is reduced for the solid Octodecane saturated rock. The measured bulk modulus of the sample seems to be completely independent of the confining pressure, whereas shear modulus still shows weak pressure dependency. The Ciz&Shapiro model prediction considerably underestimates the moduli of solid saturated rock, which suggests that stiffening occurs due to substantial reduction of compliance of grain contacts by the solid infill. This stiffening effect can be taken into account by using the solid squirt theory. However, the current solid substitution schemes considering the squirt effects give very close predictions to those by the lower embedded bound theory, which show clear discrepancies with the experimental results. Hence, we expect that only considering compliant pores might not fully describe the solid squirt effects. More study needs to be carried out in the future.

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